Gaetano Lisi and Maurizio Pugno
University of Cassino and CreaM

Tax Morale, Entrepreneurship, and the Irregular Economy

January 2011
**Tax Morale, Entrepreneurship, and the Irregular Economy**

Gaetano Lisi  
*University of Cassino and CreaM*

Maurizio Pugno  
*University of Cassino and CreaM*

**Abstract**

This paper incorporates tax morale into a search and matching model of equilibrium unemployment, with on-the-job search, extended to both the irregular sector and entrepreneurship. Tax morale is modelled as a social norm for tax compliance which renders evasion costly. The moral cost of tax evasion (the strength of the social norm) is negatively related to the fraction of entrepreneurs that evades taxes. Precisely, if the relationship is non-linear, multiple equilibria may emerge, thus accounting for differences in-between regions and countries in the size of the irregular sector. The “good” equilibrium is in fact characterised, with respect to the “bad” one, by a smaller irregular sector and a stronger tax morale.

**JEL Classification:** (A13,) E26, H26, (J24,) J64, L26, K42(, O17)

**Keywords:** tax morale; tax evasion; entrepreneurship; job search; irregular economy; shadow economy

---

* Corresponding author. Department of Economic Sciences and CreaM, University of Cassino, via S. Angelo, I-03043 Cassino (FR), Italy. Tel.: +39 0776 2994702, fax +39 0776 2994834; e-mail: m.pugno@unicas.it.
1. Introduction

Tax morale is usually defined as the intrinsic motivation to pay taxes, a moral obligation to pay taxes, a belief in contributing to society by paying taxes (see e.g. Torgler, 2007; Torgler and Schneider, 2007). The concept of tax morale was introduced in the tax compliance literature to resolve the tax compliance puzzle, i.e. to explain the high degree of tax compliance in presence, in many countries, of a very low deterrence level (Torgler, 2007; Slemrod, 2007). Indeed, traditional models à la Allingham & Sandmo (for an overview see Sandmo, 2005), based only on risk aversion, monitoring probability and expected penalty, predict far too little compliance and far too much tax evasion (Feld and Frey, 2002).

Tax morale, unlike tax evasion, does not measure individual behaviour but rather individual attitude. Hence, a high tax morale does not necessarily translate into a high level of tax compliance. However, empirical studies show the existence of a strong negative correlation between the level of tax morale and the extent of tax evasion (Torgler, 2005, for Latin America; Alm and Torgler, 2006, for the U.S. and Europe; Alm, Martinez-Vazque and Torgler, 2006, for several transition countries; and Barone and Mocetti, 2009, for Italy). Furthermore, there is evidence of a causal link of tax morale on tax evasion (Halla, 2010).

This theoretical paper incorporates tax morale into a search and matching model of equilibrium unemployment (Pissarides, 2000) with an irregular or shadow sector. Tax morale is modelled as an internalized social norm for tax compliance (Elster, 1989), or against tax evasion, which renders evasion costly (Falkinger, 1995; Kolm and Larsen, 2002; Traxler, 2010). Hence, tax evasion involves a moral cost, in the sense that an individual feels a sense of guilt or remorse for deviating from the social norm, or for defecting from others’ expectations, because s/he has not been a “good member of society” (Traxler, 2010; Kolm and Larsen, 2002). However, the more people evade taxes, the less attractive it is to follow the social norm.

In this model, the moral cost of tax evasion is negatively related to the fraction of entrepreneurs that evades taxes, and that forms the irregular sector. Precisely, if the relationship is non-linear, multiple equilibria may emerge, thus accounting for differences in-between regions and countries in the size of the irregular sector. The “good” equilibrium is in fact characterised, with respect to the “bad” one, by a smaller

---

1 Since the publication of Allingham and Sandmo’s (1972) economic model of income tax evasion, a huge number of studies have tried to find empirical support for the deterrent effect of audits and fines. The evidence, however, is weak and unstable (for a review see Kirchler et al., 2008).
irregular sector and a stronger tax morale. Therefore, this model can account for the two main shortcomings of the standard tax evasion model, i.e. Allingham and Sandmo’s (1972) model, thus explaining both the high degree of tax compliance in many countries where the level of deterrence is too low (Torgler, 2007) and the huge differences in tax compliance between countries or regions despite the same tax and punishment policies, the so-called “Palermo-Milano puzzle” (Rothstein, 2000).

The contribution of this paper is twofold. First, it introduces tax morale into a matching model of equilibrium unemployment, with on-the-job search, extended to both the irregular sector and entrepreneurship. Second, it focuses on labour demand side but works with an “equilibrium unemployment” model, thus capturing both the entrepreneurial choice and the labour market trade-off involved by the repression of irregular activity (irregular employment versus unemployment). Following the idea that both economic incentives and social norms drive individual behaviour, moral costs and search externalities clash in the entrepreneurial choice. Therefore, when an entrepreneur chooses the sector in which to create employment, s/he takes into account the moral cost as well as the start-up costs, the taxation, the monitoring probability and the expected penalty. In turn, the vacancies creation affects the probability of finding a job in both sectors.

The rest of this paper is organized as follows: section 2 presents the matching framework used in the analysis, while section 3 extents the model to include the endogenous moral cost; finally, section 4 concludes the work.

2. The model

2.1 The matching framework and workers’ search

The economy consists of a continuum of infinitely-lived individuals in the unit interval. Each individual can be either a worker or an entrepreneur. Individuals are identical in all respects except for their entrepreneurial ability \(x\), so that an individual can be an entrepreneur only if \(x > 0\), while if \(x = 0\) s/he can be a worker. We assume that an exogenous fraction \(\kappa\) of the population are endowed with positive \(x\), while the complementary fraction \(1 - \kappa\) are not, with \(1 - \kappa \geq \kappa\).

Entrepreneurs can either operate regularly or irregularly, i.e. against the tax regulations (evading taxes). Since the irregular activities are detected and repressed by

---

2 Kolm and Larsen (2002) introduce tax morale into a matching model with irregular activities but without on-the-job search and entrepreneurial choice. Traditional models of labour market focus on labour demand, whereas it is well-known that matching models focus on labour supply.
the government, and hence for the same wage flow the irregular wage is discounted at a higher rate, all workers prefer to work in the regular sector. Therefore, workers employed in irregular jobs try to move into the regular ones.\(^3\)

The matching frictions on the official side of the labour market are captured by a popular constant returns to scale (CRS) matching function (Pissarides, 2000; Petrongolo and Pissarides, 2001): \(m_r = m[v_r, u + n_s] \Rightarrow \theta \equiv \frac{v_r}{u + n_s}\), where \(\theta\) is the labour market tightness, \(v_r\) is the number of vacancies supplied by regular firms, \(u\) is the unemployment rate, and \(n_s\) is the irregular or shadow employment rate, i.e. the measure of employed job-seekers. The subscript \(i \in \{r, s\}\) denotes the sector, where \(r = \text{regular}\) and \(s = \text{shadow}\).

A crucial and novel assumption related to the workers’ preferences is introduced: only the unemployed workers who fail to find a job in the official sector search in the irregular one. Hence, this implies that matching initially takes place for official jobs and then for irregular jobs.\(^4\)

Therefore, the share of job-seekers in the irregular sector is equal to \(\bar{u} \equiv u \cdot [1 - g(\theta) dt]\), since during a short interval of time \(dt\), the unemployed worker fails to find a job in the official sector with probability \(1 - g(\theta) dt\). The instantaneous probability of finding an official job, \(g(\theta)\), has the following standard properties: \(g'(\theta) > 0\), \(g''(\theta) < 0\), and \(\lim_{\theta \to 0^+} g(\theta) = 0(\infty)\). Furthermore, we assume frictionless matching for irregular jobs, i.e. that there is a spot-market for irregular jobs.\(^5\)

The number of irregular matches is thus given by \(m_s = \min[\bar{u}, v_r]\), where \(v_r\) is the number of irregular vacancies.

As stated above, job search takes place in two sequential steps: at first, all unemployed workers search in the official sector, and afterwards (in the case of failure) they search in the irregular one. Hence, the value of searching for a job \((U)\) is given by:

---

\(^3\) Time is continuous, and individuals are risk neutral and infinitely lived. We neglect possibilities of moonlighting, so workers can perform only one activity at a time.

\(^4\) The presence of this “search path” followed by all unemployed workers is a novelty in the matching literature with respect to the standard assumptions of directed search and random search.

\(^5\) There are several other recent papers that do not consider search frictions in the shadow sector. Zenou (2008) assumes that the irregular sector is perfectly competitive; Satchi and Temple (2006) only consider the possibility of self-employment in the irregular sector, modelled as staging post for salaried work in the official one; in Albrecht et al. (2009) opportunities in the irregular sector arrive to the unemployed workers at an exogenous rate. Finally, although irregularity is not specifically addressed, Dulleck et al. (2006) assume frictionless matching for low-skilled jobs and matching frictions for high-skilled jobs. Nevertheless, for middle-income countries, at least, matching frictions in the official sector can help to explain the existence of a sizeable irregular sector, provided either that workers receive a relatively large share of the match surplus, or that recruitment costs are significant (Satchi and Temple, 2006).
\[ rU = -k + g(\theta) \cdot [W_r - U] + [1 - g(\theta)] \cdot \gamma \cdot [W_s - U] \]

where \( r \) is the discount rate; \( k \) is the exogenous search cost which reflects the search effort (implicitly assumed) of unemployed workers; \( \gamma = \min \{ u, v \} / \bar{u} \) is the probability of finding an irregular job, and \( W_i \) is the value for being employed.\(^6\)

\[ rW_s = w_s + \delta \cdot [U - W_s]; \quad rW_s = w_s + (\delta + \rho) \cdot [U - W_s] + g(\theta) \cdot [W_r - W_s] \]

where \( w_i \) is the wage rate; \( \delta \) is the exogenous job destruction rate; and \( \rho \) is the exogenous probability of a firm being discovered (and destroyed) as irregular. As in Pissarides (2000), it is implicitly assumed that employed job-seekers and unemployed workers search with the same intensity, and that they are equally good at finding official jobs. Hence, official jobs arrive to each job-seeker at the same rate \( g(\theta) \).

Finally, given the assumption of matches without frictions in the irregular sector, it may emerge the case \( v_s \geq \bar{u} \), where all unemployed workers eventually find a job in their “search path”, i.e. the outflow from the unemployment pool is exactly equal to the unemployment rate \( \frac{u - u \cdot g(\theta) dt - u \cdot [1 - g(\theta)] dt}{\gamma} \), \( \gamma \) is in fact zero if \( \gamma = 1 \). However, since the search frictions in the regular sector could cause a small (large) number of matches (unemployed), a non-trivial result requires that \( \bar{u} > v_s \).\(^7\) In this case, the probability of filling an irregular vacancy is equal to 1 and the bargaining power of workers who search for an irregular job is zero. As a result, the irregular wage is equal to a given minimum wage \( b \), with \( w_r > b \equiv w_i \).\(^8\) Therefore, a consistent equilibrium requires as a necessary condition that \( W_r > W_s > U \).

2.2 Entrepreneurship

In this section, we follow Lisi and Pugno (2010). Entrepreneurs are born with a specific and positive entrepreneurial ability \( x \) which is drawn from a known distribution, \( F : [x_{min}, x_{max}] \rightarrow [0,1] \), and affects the job productivity:

\[ rV_r = -c_r + g(\theta) \cdot [J_r - V_r]; \quad (r + \delta) \cdot J_r = xp - w_r - \tau \]

\(^6\) The search “timing” (namely, unemployed workers never search in the irregular sector before failing to find a job in the official one) implies that the two steps of the “search path” are independent. There is no positive flow of utility associated with the job search value because the search consumes the time (leisure) and resources of the unemployed worker, who does not always receive unemployment benefits.

\(^7\) This condition could remain unsatisfied when unemployment is very small, i.e. the probability of finding an official job is very large. But in this case the underground economy would be a negligible phenomenon.

\(^8\) Wages in “bad” (irregular) jobs do not depend on outside market conditions. This is a standard feature of matching models with on-the-job search, as noted by Pissarides (2000) and Boeri and Garibaldi (2002).
where $V_i$ is the value of a vacancy; $J_i$ is the value of a filled job; $c_r$ is the start-up cost; $x$ is the entrepreneurial ability; $p$ is labour productivity; while $q(\theta)$ refers to the instantaneous probability of filling a vacancy in the official sector, with $q'(\theta) < 0$, $q''(\theta) > 0$, and $\lim_{\theta \to 0^+} q(\theta) = \infty(0)$. Note that from the irregular sector standpoint on-the-job search operates as an increase in the discount rate, since it reduces the average duration of employment.\(^9\) The parameter $\phi \in (0,1)$ captures the fact that the irregular sector generally utilises less efficient technology; indeed, shadow activities are often seen as *labour intensive* activities (Busato and Chiarini, 2004).

Formal firms have to pay a lump sum tax $\tau$, whereas irregular firms do not pay taxes but sustain a moral cost, $mc$, which captures the non-pecuniary costs associated with tax evasion, i.e. the "*internal sanctions associated with emotions like guilt or remorse*" (Traxler, 2010; Elster, 1989).\(^{10}\)

A successful official match performs a net productivity equal to $(xp - \tau)$. We assume that regular wages are given by $\beta \cdot (xp - \tau)$, where $0 < \beta < 1$ is the bargaining power of workers.\(^{11}\) To ensure that regular production takes place we also assume that $(1 - \beta) \cdot (xp - \tau) > c_r$. If this did not hold true, there would be no regular jobs, which is a trivial case.

The cut-off condition, which defines a threshold level of entrepreneurial ability, $R \in [x_{min}, x_{max}]$, such that the marginal entrepreneur is indifferent to operating in the irregular or regular sector, is the following (*entrepreneurs’ indifference condition*):

$$V_r(x = R) = V_s(x = R)$$

hence, $R$ can be derived in a straightforward manner (see Appendix A):

$$R = \frac{c_r}{r + g(\theta)} + \frac{\Lambda(\theta) \cdot \tau - b + mc}{\Omega(\theta)}$$

$$R = p \cdot \left[ \frac{\Omega(\theta) - \phi}{\Lambda(\theta)} \right]$$

---

\(^9\) Alternatively, one can consider the case where $\delta$, $\rho$ or $g(\theta)$ strike, the match is destroyed but the job is not, that is, the job turns into a vacancy. This case would not change the qualitative results of the analysis.

\(^{10}\) In Kim (2003), the moral cost depends on whether evasion is detected or not. However, this assumption does not significantly change the results of the analysis.

\(^{11}\) We use the Rubinstein’s solution to a non-cooperative bargaining where it is impossible to search while negotiating. This solution, which is used by Mortensen (2005), neglects the outside options, and hence the effect of labour market tightness on wage. Indeed, as claimed by Mortensen himself (2005), the standard specification (see Pissarides, 2000) adds complexity but no further insight.
with \( \frac{q(\theta) \cdot (1 - \beta)}{(r + \delta) \cdot [r + q(\theta) + g(\theta)]} \equiv \Omega(\theta) \), and \( (1 + r) \cdot [r + \delta + \rho + g(\theta)] \equiv \Lambda(\theta) \). The restrictions which ensure the positivity of \( R \) (see Appendix A) imply that the intercept of \( V_r(x) \) is more negative than the intercept of \( V_s(x) \), and that the slope of \( V_r(x) \) is steeper than the slope of \( V_s(x) \) (see figure 1). Consequently, for \( x > R \Rightarrow V_r > V_s \), while for \( x < R \Rightarrow V_r > V_s \). This implies the following remark:

\[
== \text{Figure 1 about here (now at the end)} ==
\]

**Remark 1.** Regular jobs are managed by the more able entrepreneurs.

This key result is consistent with the standard assumption that irregular jobs are low productivity jobs (see e.g. Boeri and Garibaldi, 2002, 2006; Kolm and Larsen, 2010).

Given the c.d.f. of \( x \), i.e. \( F(x) \), then a fraction \( F(R) \) of the entrepreneurs are irregular, while a complementary fraction \( 1 - F(R) \) include regular entrepreneurs. Hence, the total number of entrepreneurs (either posting a vacancy or producing) in the irregular sector is \( \kappa \cdot F(R) = n_s + \nu_s \), while the share \( \kappa \cdot [1 - F(R)] = n_r + \nu_r \) runs a firm in the regular sector.

A key property of equation (2), which can be called the \( R - \text{curve} \), is that \( \frac{\partial R}{\partial \theta} > 0 \) (for its derivation see the Appendix A). This property captures the effect of the well-know search or congestion externalities (see Pissarides, 2000): if the ratio of hiring firms to searching workers increases, the probability of rationing is higher for the average firm. Hence, the more difficult it is to fill a regular vacancy and more entrepreneurs enter the irregular sector.

We make use of the fact that matching is pair-wise to derive an equation for the determination of \( \theta \) (see Appendix B):

\[
\theta \cdot (q(\theta) + \delta) = \kappa \cdot [1 - F(R)]
\]

This equation sets up another relationship, with respect to equation (2), between \( \theta \) and \( R \), and it will be called the \( \theta - \text{curve} \). A key property of this curve is that \( \frac{d\theta}{dR} < 0 \): intuitively, at higher \( R \) there are more irregular entrepreneurs and fewer regular entrepreneurs, so less jobs are created in the regular sector.

Therefore, equations (2) and (3) can be represented in the same diagram with axes \([\theta, R]\), as in fig. 2, and the following remark holds:

\[
== \text{Figure 2 about here (now at the end)} ==
\]

**Remark 2.** There is a unique couple of \((\theta, R)\) in this two-sector economy.
The equilibrium values of the two key variables of the model, i.e. the labour market tightness and the ability threshold for entrepreneurs to indifferently operating in one of the two sectors, can thus be obtained. In short, the model works as follows: entrepreneurs create employment in the regular or irregular sector according to the threshold level of their entrepreneurial ability, whereas unemployed workers direct their search towards the vacant jobs according to the “search path”. Hence, after their creation, the irregular vacancies are (immediately) filled once the unemployed workers enter the irregular sector and search for a job.

2.3 Unemployment rate and policy implications

The steady-state equilibrium conditions determining the employment rates \( n_r \) and \( n_i \) are given by:

\[
g(\theta) \cdot (1 - \kappa - n_i) = \delta \cdot n_i \Rightarrow n_r = \frac{(1 - \kappa) \cdot g(\theta)}{\delta + g(\theta)} \tag{4}
\]

\[
k \cdot F(R) - n_s = [\delta + \rho + g(\theta)] \cdot n_s \Rightarrow n_s = \frac{k \cdot F(R)}{[\delta + \rho + g(\theta) + 1]} \tag{5}
\]

since \((1 - \kappa - n_i) = u + n_i\), and \(\gamma \cdot \tilde{u} = v_i\). Finally, using the summing-up condition (or unemployment identity), i.e. \(1 - \kappa = u + n_r + n_i\), we obtain the unemployment rate (the Beveridge curve) of this economy:

\[
u = (1 - \kappa) - \frac{(1 - \kappa) \cdot g(\theta)}{[\delta + g(\theta)]} - \frac{F(R) \cdot \kappa}{[\delta + \rho + g(\theta) + 1]}
\]

\[
\Rightarrow \nu = \frac{(1 - \kappa) \cdot \delta}{[\delta + g(\theta)]} - \frac{F(R) \cdot \kappa}{[\delta + \rho + g(\theta) + 1]} \tag{6}
\]

note that \(\lim_{\rho \to \infty} n_s = 0\), but the steady-state unemployment rate would be higher, since \(\lim_{\rho \to \infty} u = [(1 - \kappa) \cdot \delta]/[\delta + g(\theta)]\). This result explains why governments may be reluctant to repress the irregular sector (Boeri and Garibaldi, 2006).

Furthermore, the final effect of labour market tightness on the unemployment rate in presence of an irregular sector is \textit{a priori} ambiguous, thus leading to a possible “inverse” Beveridge curve:

\[
\frac{\partial u}{\partial \theta} = -\frac{(1 - \kappa) \cdot \delta \cdot g'(\theta)}{[\delta + g(\theta)]^2} - \frac{\partial F(R) \cdot \kappa}{[\delta + \rho + g(\theta) + 1]} + \frac{F(R) \cdot \kappa \cdot g'(\theta)}{[\delta + \rho + g(\theta) + 1]^2}
\]
with \( \frac{\partial F(R)}{\partial \theta} = \frac{\partial F(R)}{\partial R} \cdot \frac{\partial R}{\partial \theta} > 0 \).

As in Lisi and Pugno (2010), the monitoring rate plays a key role in the relationship between unemployment and irregular employment. Indeed, if the monitoring rate is sufficiently large, an increase in labour market tightness (in the probability of finding an regular job) decreases both the irregular employment and the unemployment rate, so that the ‘vacancies-unemployment’ relationship (the so-called Beveridge Curve) remains negative also in presence of an irregular sector. However, an “inverse” Beveridge curve cannot be ruled out ex-ante, depending, besides the monitoring rate, on the share of irregular entrepreneurs in the economy.

### 3. Endogenous moral cost

In this section we extend the model in order to make the moral cost endogenous. The moral cost of tax evasion (i.e. the strength of the social norm) crucially and negatively depends on the share of tax evaders in the society (Gordon, 1989), the others’ compliance (Traxler, 2010), and the size of the shadow economy (Kolm and Larsen, 2002). In economies where a rather large fraction of the population is employed in the irregular sector, the moral cost is low compared to the cost of tax evasion in an economy where a rather small fraction of the population is employed in the irregular sector (Kolm and Larsen, 2002). More precisely, we assume that the moral cost is an increasing function of the size of the regular sector as follows:

\[
mc = a + b(\theta)
\]  

where \( a \) is the individual specific degree of tax morale, and \( b(\theta) \) is the non-pecuniary costs associated with emotions like guilt or remorse which depends on the fraction of irregular entrepreneurs in the population. Since \( b'(\theta) > 0 \) and \( b(0) = 0 \), and \( \lim_{\theta \to \infty} mc(\theta) < \infty \), the lower the regular sector, the more tax evasion is generally accepted, and the lower the moral cost.\(^{12}\)

Therefore, when an entrepreneur chooses the sector in which to create employment, s/he takes into account the moral cost as well as the start-up cost, the taxation, and the monitoring probability. Indeed, if the function (7) is plugged into (2), then the relationship between \( R \) and \( \theta \) may change significantly, since moral costs and

\(^{12}\) Also according to Gordon’s (1989) approach, the moral cost of tax evasion depends on both the individual specific degree of social norm internalization (exogenously given), and the fraction of evaders in the society (endogenous).
search externalities clash in the entrepreneurial choice. Hence, the $R_{-}$ curve may display decreasing segments, thus cutting the $\theta_{-}$ curve more than once. Therefore:

Remark 3. Multiple equilibria cannot be ruled out ex-ante and depend on the form of the function $mc(\theta)$.

Figure 3 depicts the possibility of three equilibria: this may occur if (7) is a function characterised by non-linearities which are typical of contagion-type diffusion, i.e. it is a logistic function. In this case, two stable equilibria emerge with an unstable equilibrium in the middle. It can thus be represented a “virtuous or vicious circle”, since tax morale affects compliance behaviour, i.e. a higher (lower) tax morale reduces (increases) the level of tax evasion (Halla, 2010), but a lower (higher) level of tax evasion, captured by the size of the shadow economy, also implies, ceteris paribus, a higher (lower) tax morale (Frey and Torgler, 2007; Halla, 2010).

Figure 3 about here (now at the end)

In short, economies with a lower tax morale can end up in an equilibrium ("bad") where the irregular sector is larger, and economies with a higher tax morale can end up in an equilibrium ("good") where the irregular sector is smaller.

Different equilibria can capture the case of some regions which exhibit a persistence of the shadow sector in very different proportions with respect to other regions, although both types of regions are characterised by a similar institutional setup. For example, the countries in the Western Europe exhibit a smaller shadow economy with respect to the countries in Eastern Europe, but also a higher rule of law and corruption perception index, which may be taken as proxies for tax morale (see table 1 at the end). An even better example is the Italy’s North-South divide, because tax and punishment policies are the same all over the country, whereas the two regions that differ in their history, i.e. in the social traditions and which persist over generations (Halla, 2010).

Our benchmark model of section 2 suggests that differences in-between regions and countries in the proportion of the irregular sector may be due to different moral costs (Halla, 2010; Alm, Martinez-Vazquez and Torgler, 2006; Torgler, 2007; Alm and Torgler, 2006; Torgler, 2005; Kolm and Larsen, 2002; Posner; 2000). Extending the

---

13 The S-shaped pattern is based on the idea of critical mass in imitative behaviour on the spatial dimension (Schelling, 1978: ch.3; Granovetter 1978).

14 With two outliers represented by the “virtuous” ex-Czechoslovakia (Czech Republic and Slovakia) and by the “corrupt” Italy. The “rule of law” is the foundation of each real democracy. Indeed, institutional arrangements, such as direct democracy, are correlated with a high level of tax morale (Alm, McClelland and Schulze, 1999; Feld and Tyran, 2002; Torgler, 2005).
model for making moral costs endogenous, as in this section, focuses the attention to the diffusion process of the standard in tax moral. This may help policy makers in finding proper interventions to increase tax moral. A bad news is that tax morale is usually regarded as very slowly-changing (Lindbeck and Nyberg, 2006; Halla, 2010), but our good news is that policy intervention may be helped by an endogenous dynamic.

4. Conclusions

This theoretical paper incorporates tax morale into a search and matching model of equilibrium unemployment, with on-the-job search, extended to both the irregular sector and entrepreneurship. Tax morale is introduced as a social norm against tax evasion (or for tax compliance) which renders evasion costly. The moral cost of tax evasion is negatively related to the fraction of entrepreneurs that evades taxes. Precisely, if it is non-linear, multiple equilibria may emerge, thus accounting for differences in-between regions and countries in the size of the irregular sector. Therefore, this model can help explain the tax compliance puzzle, i.e. why people pay taxes despite the existence of low monitoring probabilities and penalty rates or in presence of the same deterrence policies.
Appendixes

Appendix A: Properties of equation (2)

From the Bellman equations on the demand side it is straightforward to obtain:

\[ V_r = \frac{c_r}{r + q(\theta)} + \frac{q(\theta) \cdot (1 - \beta) \cdot (x_p - \tau)}{(r + \delta) \cdot [r + q(\theta)]}; \quad V_s = \frac{x_p \phi - b - mc}{(1 + r) \cdot [r + \delta + \rho + g(\theta)]} \]

hence, \( V_r = -\frac{c_r}{r + q(\theta)} + \Omega(\theta) \cdot (x_p - \tau); \quad V_s = \frac{x_p \phi}{\Lambda(\theta)} - \frac{b + \rho \cdot (\phi \tau + mc)}{\Lambda(\theta)} \)

since \( \frac{q(\theta) \cdot (1 - \beta)}{(r + \delta) \cdot [r + q(\theta)]} \equiv \Omega(\theta) \), and \( (1 + r) \cdot [r + \delta + \rho + g(\theta)] \equiv \Lambda(\theta) \). By applying equality (1), equation (2) for \( R \) can be derived.

The threshold value \( R \) is a special \( x \), so that it must be positive since \( x \geq x_{\text{min}} > 0 \).

Sufficient conditions for the positivity of \( R \) are that both the numerator and the denominator of equation (2) are positive. For labour market tightness which going to zero, this means that:

\[
\frac{(1 - \beta) \cdot (1 + r) \cdot (r + \delta + \rho)}{(r + \delta)} > \frac{b + mc}{\tau} \tag{A.1}
\]

\[
\frac{(1 - \beta) \cdot (1 + r) \cdot (r + \delta + \rho)}{(r + \delta)} > \phi \tag{A.2}
\]

since \( \lim_{\delta \to 0} \Omega = (1 - \beta) / (r + \delta) \) by the l’Hôpital rule, and \( \lim_{\delta \to 0} \Lambda = (1 + r) \cdot (r + \delta + \rho) \).

Therefore, sufficient conditions for \( R > 0 \) are that \( b, mc \), and \( \phi \) are sufficiently small, and \( \tau \) sufficiently great, and that \( \frac{\partial R}{\partial \theta} > 0 \). Sufficient conditions for \( \frac{\partial R}{\partial \theta} > 0 \) are that the numerator of equation (2) is increasing in \( \theta \), while the denominator is not. Indeed,

- the numerator of \( R \) is increasing in \( \theta \) if:

\[
\frac{\partial}{\partial \theta} \left\{ \frac{c_r}{r + q(\theta)} + \Omega(\theta) \cdot \tau - \frac{b + mc}{\Lambda(\theta)} \right\} = -\frac{q'(\theta) \cdot c_r}{[r + q(\theta)]^2} + \Omega'(\theta) \cdot \tau + \frac{\Lambda'(\theta) \cdot (b + mc)}{[\Lambda(\theta)]^2} > 0,
\]

where \( \Omega(\theta) = \frac{q'(\theta) \cdot (1 - \beta) \cdot (r + \delta) \cdot [r + q(\theta)]}{(r + \delta) \cdot [r + q(\theta)]^2} < 0 \), and \( \Lambda'(\theta) = (1 + r) \cdot g'(\theta) > 0 \), i.e. it is sufficient that \( c_r \cdot (r + \delta) \geq \tau \cdot (1 - \beta) \cdot r \). This is a realistic restriction since \( \beta \) is a substantial fraction (usually \( \beta = 0.5 \));

- the denominator of \( R \) is not increasing in \( \theta \) if:

\[
\frac{\partial}{\partial \theta} \left\{ \frac{\phi}{\Lambda(\theta)} \right\} = \Omega'(\theta) + \frac{\Lambda'(\theta) \cdot \phi}{[\Lambda(\theta)]^2} \leq 0 \Rightarrow \frac{q'(\theta) \cdot (1 - \beta) \cdot (r + \delta) \cdot [r + q(\theta)]}{(r + \delta) \cdot [r + q(\theta)]^2} + \frac{(1 + r) \cdot g'(\theta) \cdot \phi}{(1 + r) \cdot [r + \delta + \rho + g(\theta)]^2} \leq 0.
\]
This condition can be easily proved if the properties of matching function in the usual Cobb-Douglas form are applied:\(^\text{15}\)

\[
-\alpha \cdot \theta^{-\alpha} \cdot (1 - \beta) \cdot (r + \delta) \cdot r + \frac{(1 + r) \cdot (1 - \alpha) \cdot \theta^{-\alpha} \cdot \phi}{\left[(r + \delta) \cdot (r + \theta^{-\alpha})\right]^2} \leq 0
\]

being \(m(\theta^{-1}, 1) \equiv q(\theta) = \theta^{-\alpha}\), and \(m(1, \theta) \equiv g(\theta) = \theta \cdot q(\theta) = \theta^{1-\alpha}\). The condition can thus be rewritten as

\[
\frac{(1 - \beta) \cdot (r + \delta) \cdot r}{\left[(r + \delta) \cdot (r + \theta^{-\alpha})\right]^2} - \frac{(1 + r) \cdot (1 - \alpha) / \alpha \cdot \theta \cdot \phi}{\left[(1 + r) \cdot (r + \delta + \rho + \theta^{1-\alpha})\right]^2} \equiv \Gamma(\theta) \leq 0,
\]

with \(\lim_{\theta \to 0} \Gamma(\theta) = 0\). Therefore, since \(0 < \theta < \infty\), then a sufficiently small \(\phi > 0\) (as already stated above) ensures that the denominator of \(R\) is not increasing in \(\theta\).

Finally, \(\lim_{\theta \to 0} R \equiv \alpha > 0\), by conditions (A.1) and (A.2); and \(\lim_{\theta \to \infty} R = \infty\), since \(\lim_{\theta \to \infty} \Omega = 0\), and \(\lim_{\theta \to \infty} \Lambda = \infty\). Note that \(\alpha < x_{\text{max}}\), since equation (2) has been built for \(R \in [x_{\text{min}}, x_{\text{max}}]\).

**Appendix B: Properties of equation (3)**

The evolution of employment \(n\) can be expressed in terms of both worker’s transition rates and firm’s transition rates (see Fonseca et al. (2001) and Pissarides (2002)),

\[
\dot{n}_r = \left[1 - \kappa - n_r \right] \cdot g(\theta) - \delta \cdot n_r,
\]

\[
\dot{n}_r = \{\kappa \cdot [1 - F(R)] - n_r \} \cdot q(\theta) - \delta \cdot n_r,
\]

Hence, in steady-state \((\dot{n}_r = 0)\), we get:

\[
n_r = \left[1 - \kappa \right] \cdot g(\theta) \over g(\theta) + \delta
\]

\[
n_r = \frac{\kappa \cdot [1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}
\]

From (B.1) and (B.2), it follows that for any level of employment it must be true that:

\[
\frac{[1 - \kappa] \cdot \theta \cdot q(\theta)}{g(\theta) + \delta} = \frac{\kappa \cdot [1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}
\]

\[
\Rightarrow \frac{g(\theta) + \theta \cdot \delta}{g(\theta) + \delta} = \frac{\kappa \cdot [1 - F(R)]}{[1 - \kappa]} (B.3)
\]

By the properties of the matching function and because of the restriction \(\kappa / (1 - \kappa) \leq 1\), the left-hand side of (B.3) is increasing in \(\theta\); whereas, the right-hand side of (B.3) is decreasing in \(R\). Therefore, total differentiation of equation (B.3) gives a negatively sloping relation between \(\theta\) and \(R\). As \(R\) tends to \(x_{\text{max}}\), \(\theta\) tends to 0, since \(F(R)\) tends to 1; whereas, a positive level of \(\theta = \tilde{\theta} > 0\) as \(R\) tends to \(x_{\text{min}}\) is obtained by the fixed point theorem. In fact, if \(R\) tends to \(x_{\text{min}}\), then \(F(R)\) tends to 0, so that:

\[
g(\theta) + \theta \cdot \delta = \frac{\kappa}{(1 - \kappa) \cdot (g(\theta) + \delta)} (B.4)
\]

Note that for \(\theta = 0\) the intercept of the r.h.s. of (B.4) is higher than the intercept of the l.h.s., while the slope of the l.h.s. of (B.4) is steeper than the slope of the r.h.s.: hence, a unique and positive \(\theta\) exists when \(R\) tends to \(x_{\text{min}}\).

\(^{15}\) Surveying the empirical evidence, Petrongolo and Pissarides (2001) summarize the wealth of support for a Cobb-Douglas matching function with constant returns to scale.
References


Figures and Table

**Figure 1. Entrepreneurs’ indifference condition**

**Figure 2. Equilibrium interior solution**
Figure 3. Multiple equilibria
Table 1. Data Appendix

<table>
<thead>
<tr>
<th>European countries</th>
<th>Rule of Law index *</th>
<th>Shadow Economy (% of GDP) **</th>
<th>Corruption Perception index ***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>99</td>
<td>9.8</td>
<td>7.90</td>
</tr>
<tr>
<td>Belgium</td>
<td>89</td>
<td>22.5</td>
<td>7.10</td>
</tr>
<tr>
<td>Denmark</td>
<td>99.5</td>
<td>18.2</td>
<td>9.30</td>
</tr>
<tr>
<td>Finland</td>
<td>97.6</td>
<td>18.5</td>
<td>8.90</td>
</tr>
<tr>
<td>France</td>
<td>90</td>
<td>15.4</td>
<td>6.90</td>
</tr>
<tr>
<td>Germany</td>
<td>93.3</td>
<td>16.1</td>
<td>8.00</td>
</tr>
<tr>
<td>Ireland</td>
<td>94.3</td>
<td>16</td>
<td>8.00</td>
</tr>
<tr>
<td>Italy</td>
<td>62.2</td>
<td>27.2</td>
<td>4.30</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>96.2</td>
<td>9.9</td>
<td>8.20</td>
</tr>
<tr>
<td>Malta</td>
<td>91.4</td>
<td>27</td>
<td>5.20</td>
</tr>
<tr>
<td>Netherlands</td>
<td>94.7</td>
<td>13</td>
<td>8.90</td>
</tr>
<tr>
<td>Norway</td>
<td>100</td>
<td>19.5</td>
<td>8.60</td>
</tr>
<tr>
<td>Portugal</td>
<td>83.7</td>
<td>22.5</td>
<td>5.80</td>
</tr>
<tr>
<td>Spain</td>
<td>85.2</td>
<td>22.9</td>
<td>6.10</td>
</tr>
<tr>
<td>Sweden</td>
<td>98.1</td>
<td>19.6</td>
<td>9.20</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>92.3</td>
<td>12.9</td>
<td>7.70</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>51.2</td>
<td>38.5</td>
<td>3.80</td>
</tr>
<tr>
<td>Cyprus</td>
<td>84.2</td>
<td>29.4</td>
<td>6.60</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>77</td>
<td>19.8</td>
<td>4.90</td>
</tr>
<tr>
<td>Estonia</td>
<td>84.7</td>
<td>40.3</td>
<td>6.60</td>
</tr>
<tr>
<td>Greece</td>
<td>73.2</td>
<td>29.9</td>
<td>3.80</td>
</tr>
<tr>
<td>Hungary</td>
<td>76.1</td>
<td>25.8</td>
<td>5.10</td>
</tr>
<tr>
<td>Latvia</td>
<td>71.3</td>
<td>41.7</td>
<td>4.50</td>
</tr>
<tr>
<td>Lithuania</td>
<td>67.5</td>
<td>31.9</td>
<td>4.90</td>
</tr>
<tr>
<td>Poland</td>
<td>65.1</td>
<td>28</td>
<td>5.00</td>
</tr>
<tr>
<td>Romania</td>
<td>53.6</td>
<td>36.3</td>
<td>3.80</td>
</tr>
<tr>
<td>Slovakia</td>
<td>67</td>
<td>19.7</td>
<td>4.50</td>
</tr>
<tr>
<td>Slovenia</td>
<td>82.3</td>
<td>28</td>
<td>6.60</td>
</tr>
<tr>
<td>Turkey</td>
<td>55.5</td>
<td>32.9</td>
<td>4.40</td>
</tr>
</tbody>
</table>

