Theoretical foundations of pay-as-you-go defined-contribution pension schemes

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Abstract

The paper inquires into Notional Defined Contribution pension schemes, which retain the pay-as-you-go financing method while adopting the award and indexation formulas typical of funded, defined-contribution systems. It inquires the properties of the new arrangement and compares its theoretical foundations with the literature that, between the 1955 and 1966, identified the implicit return granted by unfunded systems.

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1 Introduction

It is common knowledge that funded pension systems are of two types. The Defined- Contribution (DC) scheme provides contributions-related benefits. It pays out to each participant a flow of annual instalments equivalent to the contributions previously paid in (gross of the interests matured). The Defined-Benefit (DB) scheme provides earnings-related benefits. It allows for contributions paid in by workers with flat wage-profile being paid out, in part, to workers with rising wage-profile.

Traditionally, the DB scheme has been the sole arrangement for pay-as-you-go (PAYG) pension systems. As for funded systems such a scheme tend to produce some redistribution.

In the nineties, the range of possible pension plans was extended by a sort of ‘genetic innovation’ that inspired the pension reforms implemented in Italy (1996) and Sweden (1998)\(^1\) and that is strongly influencing social security debates now taking place in other European countries.\(^2\) Retaining the PAYG financial architecture, meaning that current pension expenditure is still financed by current contribution revenues, the new plan brings in the equivalence of benefits and contributions drawn from the DC, funded scheme. This is why in Italy it is called the ‘contributions-based’ plan. Internationally, different terms are more usual such as ‘notionally funded’, ‘notional personal accounts’, ‘notional defined contribution’ (NDC).\(^3\)

\(^1\) Similar reforms have been implemented also in Latvia (1996) and Poland (1999).

\(^2\) Including Austria, Czech Republic, France and Germany. For a survey of the debates see Holzmann and Palmer (2006).

\(^3\) The scheme was firstly developed in the early 1990s in Italy and in Sweden independently. In Italy, it has been proposed by Gronchi with the aim to remedy the unfair redistribution typical of the earnings-based formulas and to ensure sustainability for the PAYG system. A different version was put forward by Niccoli with the alternative aim to award the forced savings accrued to social security with
The present paper examines the properties and theoretical bases of the PAYG-DC scheme while comparing them with the main features of the traditional PAYG-DB scheme in the light of the well known findings of Samuelson (1958) and Aaron (1966) that recognized the wage bill growth rate as the internal rate of return (IRR) to contributions in a steady growing economy.\(^4\)

Section 2 examines the PAYG-DB scheme. It also presents a reformulation of the Samuelson-Aaron theorem for an economy where individuals are heterogeneous and pension benefits are earnings-related. This context is crucial in order to remark that in the DB scheme the wage bill growth rate is the ‘generational’ IRR the same return earned by savings directed towards financial markets. In particular, see Niccoli (1992), Italian Ministry of Treasury (1994) of which Gronchi (1993) is the first draft while Gronchi (1995 b) is a revised version, Gronchi (1994 a, b), Niccoli (1994) and Gronchi (1995 a). The attention of the Italian Parliament was drawn to the new ideas in 1994 when a detailed contributions-based plan was produced by the Democratic Party of the Left. One year later, the same ideas were resumed by the draft bill that the Dini Cabinet brought before Parliament and that was approved by a large majority of votes.


This mentioned Italian and Swedish literature of the early 1990s is unaware of the anticipations to be found in two important works of the late 1960s, rediscovered in Gronchi (1998) and Valdés-Prieto (2000), one of which is by J. Buchanan and the other by O. Castellino. In the scheme proposed by Buchanan (1968), all income earners are requested to purchase “social insurance bonds” whose return is set equal to the higher of the rate of growth in GNP and the rate of interest on U.S. treasury bonds. Castellino (1969) thoroughly analyzed the properties of a mixed PAYG scheme, where only part of the benefit is contributions-related and the return is set equal to the growth of the average wage. Valdés-Prieto (*ibidem*) recalls that an old-age contributions-based scheme was also outlined by Boskin *et al.* (1988) within a more comprehensive social security model entirely based on personal accounts.\(^4\)

\(^4\) The same results had already been obtained by De Finetti (1956) in a paper whose ‘impact factor’ was very low.
remunerating overall contributions paid in by a whole cohort and not individual contributions paid in by cohort members. It is finally shown how individual IRRs depend on wage profile and retirement age.

Section 3 analyzes the PAYG-DC scheme. Calculation and indexation formulae for the contributions-related benefits are identified that allow to reward all participants with the same annual, explicit (not internal) rate of return. It is then argued that such benefits are sustainable (can be wholly financed by any given contribution rate) if and only if the uniform return is set equal to the wage bill growth rate. An important implication of this new Samuelson-Aaron type theorem is that in DC schemes the contribution rate is no longer deputed to guarantee balance; it is rather deputed to determine benefit levels. The theorem holds for semi-steady economies where the wage bill growth rate is allowed to vary as a consequence of variable wage growth rates. Constancy of the employment growth rate is still needed\(^5\).

Section 4 points out some concluding remarks.

## 2 The PAYG-DB scheme

### 2.1 Award and indexation formulae

In PAYG-DB schemes, the pension benefit is earnings-based in that the first instalment is the product of three factors: the number of years of contributions, a percentage called the ‘accrual rate’, and a ‘conventional salary’ equal to the mean of

\(^5\) In his interesting analysis of PAYG-DC schemes sustainability, Valdés - Prieto (2000) seems to ignore that variable wage growth rates cannot affect balance.
the earnings obtained over the final (possibly all) years of work, revalued at a given rate. More exactly, assuming for simplicity that time is discrete, and letting

- $1, \ldots, n$ denote the years in which the worker pays in contributions, so that $n + 1$ designates the year in which the first pension instalment is paid out;
- $w$ be the annual wage;
- $k$ be the accrual rate;
- $r$ be the number of final annual salaries taken into account in calculating the conventional salary,
- $\gamma$ be the revaluation rate,

the award formula for the first annual benefit is:

$$
p = n \cdot k \cdot \frac{\sum_{i=n-r+1}^{n} w_i \prod_{j=i+1}^{n+1} (1 + \gamma_j)}{r}.
$$

Despite the general tendency to bring $r$ closer to $n$, a large part of the OECD countries and the majority of developing countries still have $r < 10$. So one can say that in reality pension formulae are still dominated by ‘last earnings rules’.

Traditionally indexed to salaries, earnings-based pensions are now increasingly indexed to prices only, given the growing budget difficulties produced by incipient population aging.

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6 In some countries the average is calculated over a certain number of ‘top’ earnings rather than the last ones. Of course, the two options are equivalent when the salary grows by age. The conventional salary is sometimes split into brackets the higher of which are associated with lower accrual rates. Progressiveness is thus ensured.

7 See Disney (1999).
2.2 Individual versus generational internal rate of returns

Following Samuelson (1958), Aaron (1966) proved that in a steady economy where wages and employment grow at constant rates, a DB scheme that adjusts the contribution rate at the level which guarantees balance, offers to each participant an IRR on contributions equal to the growth rate of the wage bill. Aaron’s theorem relies on the assumption that agents are homogeneous.\(^8\)

If one allows for heterogeneity, individual IRRs prove to be no longer equal and the wage bill growth rate turns into a ‘generational’ IRR remunerating overall contributions paid into the system by a cohort as a whole. More precisely, it becomes the rate of interest that sets equal to zero the present value of the cash-flows constituted by the aggregate yearly contributions paid by a cohort and the aggregate yearly pension benefits received.

The generational nature of such a return prevents it from measuring profitability for cohort members to participate in a DB scheme. Rather, this is gauged by IRRs of cash-flows constituted by individual contributions and pension benefits. Analysis of individual IRRs shows redistribution phenomena that are mostly regressive. In particular, DB schemes reward early retirement and rising wage profiles. Flat careers are penalized, typically blue-collar and clerical workers. These phenomena can be attenuated but not eliminated by making \(r\) tend to \(n\) within formula (1).

All these aspects are analysed in section A.1 of the Appendix.

\(^8\) In particular, Aaron assumes that working life as well as pension duration (though different from each other) are the same for all individuals. He also assumes that all workers receive the same salary and that all retirees receive a pension equal to this (unique) salary.
3 The PAYG-DC scheme

The disparity in individual IRRs can be offset by a scheme that conceives the PAYG system as a ‘virtual bank’ where individuals have ‘personal accounts’ in which the contributions are ‘deposited’ and from which the pension benefits are ‘withdrawn’.

3.1 Calculating and indexing pension benefits

The first annual benefit (still denoted as $p$) must satisfy the constraint (2)

$$a \cdot \sum_{i=1}^{n} w_i \cdot \prod_{j=i+1}^{n+1} (1 + \pi_j) = p \left( 1 + \sum_{i=n+2}^{n+m} \prod_{j=n+2}^{i} \frac{1 + \sigma_j}{1 + \pi_j} \right)$$

where $\pi_j$ denotes the value that the return granted by the virtual bank to all accounts takes in year $j$; $\sigma_j$ denotes the value that the benefits (annual withdrawals) indexation rate takes in year $j$; $a$ denotes the (constant) contribution rate; $m$ is life expectancy at retirement while $n$ still designates the duration of the working life. The first member of (2) is the worker’s claim on the virtual bank at retirement (the total value of contribution payments including returns matured) while the second is the present value of future withdrawals (pension benefits).

Constraint (2) formalizes the ‘one euro for each euro’ principle in that it ensures that benefits exactly exhaust contributions. In other words, it guarantees that the last withdrawal (annual benefit) brings the virtual account balance to zero.

From equation (2) it follows

$$p = \left[ a \cdot \sum_{i=1}^{n} w_i \cdot \prod_{j=i+1}^{n+1} (1 + \pi_j) \right] \cdot h$$

where
Adopting Italian terminology, hereafter $h$ is referred to as ‘conversion coefficient’ and the term in square brackets on the right-hand side of (3) as ‘notional capital’ (accrued at retirement).\(^9\)

At retirement the notional capital is obviously known, whereas calculating the conversion coefficient would require knowledge of the values that both the indexation rate ($\sigma$) and the rate of return ($\pi$) will take in the next $m$-1 years. Nevertheless, the conversion coefficient can be calculated if one forgoes the idea of choosing the indexation rate independently from the rate of return and agrees to peg the former to the latter through the equation

\[
\sigma_j = \frac{1 + \pi_j}{1 + \delta} - 1, \tag{5}
\]

where $\delta$ is a ‘deviation rate’, to be chosen by the policy maker, between the rate of return and the rate of indexation.\(^{10}\) Substituting (5) into (4), the conversion coefficient takes the form

\[
h = \left[ \sum_{i=1}^{m} (1 + \delta)^{-i} \right]^{-1} \tag{6}
\]

thus reducing to a function of two independent variables: the deviation rate and life expectancy ($m$).

\(^9\) In Swedish practice, the pension award is obtained by dividing the first term in Equation (2), which is called the ‘account balance’ by the reciprocal of $h$, called the ‘annuitization divisor’ (National Social Insurance Board 2002, p. 61).

\(^{10}\) The Italian DC reform took $\delta = 1.5\%$, Swedish took $\delta = 1.6\%$. These high values of $\delta$ are intended to provide standard workers with replacement rates that are similar to those in being prior to the DC reforms.
Equations (3) and (6) define the newly granted annual benefit (first annual instalment).

In the relevant (economically meaningful) domain $\delta > -1$ the graphs of (5) and (6) are as in Figure 1. They show that choosing higher values for $\delta$ does not only imply higher conversion coefficients, hence more generous initial pensions, but also less generous indexation rates, giving rise to a trade-off between the two.\(^{12}\)

$$\sigma_j = \frac{1+\pi_j}{1+\delta} - 1$$

$$h = \left[ \sum_{i=1}^{\infty} (1+\delta)^{-i} \right]^{-1}$$

Figure 1

Note two implications following from Figure 1:

- when $\delta = 0$ the indexation rate equals the rate of return, while the first annual benefit is obtained by dividing the notional capital by life expectancy at retirement;

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\(^{11}\) Outside such a domain either $\sigma_j$ vanishes (for $\delta = -1$) or $\sigma_j < -1$ (for $\delta < -1$).

\(^{12}\) The value chosen for $\delta$, and resulting value for $\sigma$, do not affect uniformity of individual returns. They rather determine the benefit time pattern. Nevertheless, when choosing high values for $\delta$, one should take into account the ‘social’ problem deriving from the indexation rate of pensions possibly being much smaller than wages growth (determining the newly granted pensions growth). Such a divergence causes ‘vintage pensions’, i.e. differences among coexistent benefits started in different years.
• as $\delta$ increases, the indexation rate approaches –1 while $h$ approaches 1 so that no annual benefit can follow the first one, which turns into a lump sum exhausting the entire notional capital.

Formula (6) also shows that the conversion coefficient decreases as life expectancy increases, so that (for any given $\delta$) it increases with retirement age.

Calculating the first annual benefit on the basis of formulae (3) and (6), and indexing it according to formula (5), ensures that each year $j$ all virtual accounts of both workers and pensioners are evenly remunerated with the value $\pi_j$ that the chosen rate of return takes in that year.

3.2 Guaranteeing sustainability

If all one asked of the DC scheme were uniformity of individual returns within and across generations, then all one would need is to set $\pi$ at a constant value in equations (3) and (5). But one might also aim to guarantee sustainability. Note that this cannot be pursued by adjusting the contribution rate since higher (lower) contribution revenues would be automatically followed by higher (lower) benefits spending. The balance of a DC scheme can rather be obtained by properly setting the rate of return.

Let us now consider the following rate:

$\pi_j = (1 + \alpha_j) \cdot (1 + \lambda_j)^{-1}$

where $\alpha$ and $\lambda$ denote wages and employment growth rates respectively, so that compounded rate (7) is the wage bill growth rate. It can be proved that substituting (7) into (3) and (5) keeps annual spending in line with annual contributions, whatever the (constant) contribution rate.
Note that this new Samuelson-Aaron type theorem holds under the assumption that \( \lambda \) is constant with respect to \( j \) while it does not require that \( \alpha \) be also constant.

The proof is given in section A.2.1 of the Appendix for the same economy (with heterogeneous agents) posited in section A.1.1. The theorem authorizes to designate the wage bill growth rate as the ‘sustainable return’ of PAYG-DC schemes.\(^{13}\)

It should be emphasized that the wage bill growth rate plays an important role both in DB and DC PAYG schemes. Nevertheless, in DB schemes it is no more than a benchmark for measuring gains and losses entailed by different individual IRRs, whereas in DC schemes it is explicitly credited each year to all personal accounts. The analysis carried out in this section allows to point out a further, salient, difference. In DB schemes the wage bill growth rate is implied by the system’s balance (more exactly by fixing the contribution rate at the level which guarantees the balance) provided that the economy is fully steady, i.e. both wages and employment grow exponentially; in DC schemes the wage bill growth rate, annually credited to personal accounts, is itself the cause of system’s balance, provided that the economy is semi-steady, i.e. only wages grow exponentially.

### 3.3 Properties and cautions

Equations (3), (5), (6) e (7) define a PAYG scheme where:

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\(^{13}\) In neither Italy nor Sweden has \( \pi \) been set equal to such sustainable rate. Italy took GDP growth, Sweden average wage growth. GDP growth tends to be sustainable if the distributive shares tend to be stable over time. The average wage growth tends to be sustainable if employment tends to be constant. As a consequence of the declining population presaged by demographic projections, in the decades to come the rate of return chosen by Sweden will tend to exceed the sustainable rate. In 2001 the Swedish Parliament acted to remedy this threat, introducing an ingenious balance mechanism automatically lowering the rate of return when long run unbalances are envisaged. To this respect, see Settergren (2001). The mechanism went into effect in 2003.
• intra-generational uniformity of individual IRRs is (sufficiently) assured in so far all members of each cohort have their accounts credited with the same pattern of annual rates of return;\(^{14}\)

• since one can assume that the wage growth rate \(\alpha\) tend to be constant in the long run, (sufficient) inter-generational uniformity of individual IRRs is assured as well, provided that also the employment growth rate \(\lambda\) is (sufficiently) constant in the long run;

• under the latter hypotheses, sustainability is (sufficiently) assured too (even though \(\alpha\) should be variable over time).

Against these properties, some cautions must be taken.

3.3.1 Heterogeneous mortality

One should be aware of the ‘actuarial’ nature of constraint (2), whose second term unavoidably includes the \(m\) yearly pension instalments that can be foreseen at retirement rather than their actual number that will have been assessed by the end of the retiree’s life. For that reason, contributions (plus the returns matured) are exactly given back to the ‘representative’ retiree only. The longer-lived will over-withdraw their contributions, the shorter-lived will under-withdraw. As a result, intra-generational uniformity of individual IRRs is jeopardized \textit{ex post}.

Nevertheless uniformity would still be assured \textit{ex ante} if all workers electing to retire at a given age had the same life expectancy. Unfortunately, this is not the case. For example, the rich are longer-lived than the poor, women longer-lived than men,

\(^{14}\) The PAYG-DC scheme (like funded schemes where the return equals the market interest rate) cannot avoid negligible intragenerational disparities in IRRs due to differences in individual cash-flows of contributions and benefits. One should also note that such disparities are random rather than unfairly associated with wage profiles and retirement ages as in DB schemes.
married people longer-lived than single, rural residents longer-lived than city-dwellers.

The mentioned correlation between life expectancy and income, hence between the duration of the pension and the size of the notional capital, undermines also sustainability (that DC schemes can assure under constancy of \( \lambda \)). Infact each year the benefits ‘gained’ by the longer-lived exceed those ‘lost’ by the early deceases.\(^{16}\)

The consequences of heterogeneous mortality on both the uniformity of individual IRRs and sustainability could be sterilized by diversifying the conversion coefficients by homogeneous social groups; but diversification would be technically difficult and its social acceptability could not be taken for granted.

3.3.2 Declining fertility

Working age population, therefore employment, may follow cyclical path (rather than exponential) due to fertility changes\(^ {17}\). In nowadays demographic slump, the return defined by equation (7) tend to decrease year by year, thus precluding IRRs intergenerational uniformity.

Decreasing returns are PAIG-DC response to fewer workers having to sustain one pensioners, i.e. to produce enough ‘food’ for him and for themselves. Could the

\(^{16}\) Actually, this imbalance might be offset by some contrasting correlations, such as that between longevity and gender in so far women, who are longer-lived, also have lower salaries.

\(^{17}\) Mortality changes do not affect working age population. Therefore they do not influence the rate of return defined by equation (7). They rather affect conversion coefficients defined by equation (6).

\(^{19}\) Nor could this problem be easily avoided by letting indirect taxes (instead of contributions) finance the pension system. Enlarging the base (GDP instead of wage bill) could only offer a temporary remedy since the new base should have the same long run pattern.
financial architecture of funded systems overcome the economic problem which is constituted by such a lack of resources?  

3.3.3 Who really pays the contributions?

A caution is also called for on the assumption (implicitly made by DC approach) that the contributions are paid entirely by workers, even though in all countries social security taxes are shared between workers and employers. If this formal-legal division were also one of economic substance, i.e. if employers’ contribution rate were not shifted back, the DC scheme would fail the IRRs-uniformity test. For if \(a_l\) denotes the tax contribution rate for labourers and \(a_e\) that for employers, the notional capital would be

\[
M = (a_l + a_e) \cdot \sum_{i=1}^{n} w_i \cdot (1 + \pi)^{n+1-i}
\]

\[
= \sum_{i=1}^{n} a_l \cdot w_i \cdot (1 + \pi)^{n+1-i} \cdot \left[1 + \left(\frac{a_e}{a_l} \cdot \frac{\sqrt[1+n-i]{1+\frac{a_e}{a_l}} - 1}{\sqrt[1+n-i]{1+\frac{a_e}{a_l}}}ight) \right]^{n+1-i}
\]

showing that worker’s contributions would be remunerated not only according to the return defined by equation (7), for simplicity here assumed to be constant, but also according to an additional return implicitly generated by the employer’s contribution. The algebraic expression for this extra-return shows that it increases as the difference between \(n\) and \(i\) decreases. Consequently, it would reward early retirements for which on the average \(n\) is nearer to \(i\), and also penalize careers with slow earnings growth for which the last yearly contributions, better remunerated as \(i\) is closer to \(n\), count less heavily. The same kind of unfairness proper to DB schemes would be reproduced.

The creation of the first social security systems in Europe and in the United States in the late nineteenth and early twentieth centuries gave rise to an initial body
of literature that basically assumes employers shift the cost of social security contributions on to workers. The recent literature distinguishes between general and benefit-linked payroll taxes. As to the former, a partial shift onto prices is admitted, with distortions caused by differing individual propensities to consume. As to the latter, the conclusion is reached that workers accept the complete incidence on wages insofar as they have confidence in the actuarial equivalence between contributions and benefit. Since such equivalence is perfectly implemented by the DC scheme, complete incidence on wages should be admitted. Full incidence on wages is also confirmed by most empirical studies. But in case these theoretical and empirical

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20 The problem is raised again by Vitaletti (2000).

21 Harris (1941, pp. 285-86), for instance: “The economists who, in the years preceding the introduction of the Social Security Act, had given the problem of incidence careful consideration, seem to have been in general agreement that a payroll tax, whether levied on the workers or the employer, would be paid ultimately by the workers. [...] In the years that have passed since the Social Security Act became law, the weight of informed opinion seems to be that the payroll tax is borne largely by the workers”. Authors maintaining that these contributions weigh exclusively on wages include Brown (1922), Hall (1938), Meriam (1933), Bauder (1936), Castellino (1969). To be sure, there are also those who argue that the cost is transferred partly to prices as well as to wages: Bargoni (1968), Breack (1953), Conrad (1954), Cosciani (1961), Dalton (1954), Fuà (1965), Harris (1941), Kimmel (1950), Musgrave et al. (1951), Richardson (1960), Taylor (1953). All categorically rule out any incidence on profit.

22 See Blinder et. al (1980), Gordon (1983), Burkhauser and Turner (1985), Browning (1985). However, as there is a very large overlap between the working and consuming populations, the thesis of 100% incidence on wages (albeit through prices) prevails. The incidence on wages is ruled out only on the hypothesis that firms pay efficiency wages (Pisauro, 1991).

23 For partial equilibrium analyses, see Browning (1975), Summers (1989); for a general equilibrium analysis, Auerbach and Kotlikoff (1985). Alesina and Perotti (1997, pp. 922-3) argue that the equivalence between the contributions paid and the benefit entitlement is differently perceived depending on the institutional characteristics of the labor market. In particular the perception of the equivalence would be stronger for those countries where wages are centrally negotiated.

24 See also Feldstein (2002, p.7). Note that insofar as the DC scheme makes people not to perceive contributions as a tax it is also able to counter social security tax evasion.

conclusions should not be always and totally justified, one must be aware of the consequences on uniformity of returns on employees’ contributions.

3.3.4 Declining mortality

The balance of the DC scheme may be threatened by decreasing mortality which prevents actual expected life at retirement, entering into the calculation of the conversion coefficient, from being sufficiently approximable by the one which is indicated in the most recent mortality table. Underestimates of expected life would cause the violation of constraint (2). What happens is that benefits are protracted beyond the limit envisaged in calculating the conversion coefficient.

The remedy cannot be the use of ‘forward looking’ conversion coefficients (based, that is, on projected mortality) instead of ‘backward looking’ ones (based on observed mortality). In fact forward looking rates tend to overshoot, i.e. to produce surpluses, because newly granted benefits would be cut before their extra duration was realized.\(^\text{27}\) This circumstance suggests that, in the presence of decreasing mortality, sustainable return is higher than wage bill growth.\(^\text{28}\)

3.4 Stability of the balance

We now want to inquiry the ability of the DC scheme to restore balance when a permanent shock occurs in the employment growth rate. Indeed an increase in \(\lambda\), generates temporary surpluses while a reduction results in temporary deficits. However, the simulation developed in section A.2.2 (for the case of an increase in \(\lambda\) )

\(^{27}\) See Valdés-Prieto (2000, pp.413-14). Note that conversion coefficients based on mortality projections are debatable raising major problems of social acceptability.

\(^{28}\) See Settergren and Mikula (2006).
shows that a new balance is spontaneously reached. The DC scheme is thus endowed with an ‘automatic pilot’. After a (permanent) reduction in $\lambda$, the pilot timely takes countermeasures to curb expenditure.

Note that this prompt intervention fairly takes the twofold form of slower indexation of pensions in being and smaller returns to workers’ notional capitals being formed, thus containing future benefits. In such a way, all existing cohorts share the burden of adjustment.

3.5 The non-equivalence of parametric reforms

One could argue that a parametric reform of the DB scheme extending the award formula to the whole career is a quick way of attaining the purpose of the DC scheme, while avoiding the announcement of falsely innovative reforms. Actually, the argument can be easily denied. In fact, for $r = n$ equation (1) becomes

$$p = k \cdot \sum_{i=1}^{n} w_i \cdot \prod_{j=i+1}^{n+1} (1 + \gamma_j),$$

from which it follows

$$p = \left[ a \cdot \sum_{i=1}^{n} w_i \cdot \prod_{j=i+1}^{n+1} (1 + \gamma_j) \right] \cdot \frac{k}{a}.$$

Indeed, the earnings-based pension (9) displays analogies with the contribution-based one defined by equations (3) and (6), as it too is the product of a ‘notional capital’ and a ‘conversion coefficient’. But these analogies are merely

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29 These results differ from those of Valdés-Prieto (2000, pp. 407-08), who seems to maintain that increases in $\lambda$ generate temporary deficits.

30 For instance, see Chicon (1999). This erroneous belief was one source of the resistance in Italy to the contributions-based reform of 1996. The reform was said to be useless, in that $r$ had been set equal to $n$ in 1992.
formal, since a real DC scheme can be obtained only by choosing specific values of the parameters involved in formula (9). In particular, it is necessary:

- to set $\gamma$ equal to the wage bill growth rate,
- to differentiate the accrual rate $k$ by age according to the equation $k = h \cdot a$.

Moreover DC schemes require that indexation be endogenously defined by equation (5).

4 Conclusions

The comparative examination of the PAYG-DB and PAYG-DC schemes presented in sections 2 and 3 shows that:

- the former aims to achieve uniformity of replacement rates by awarding pension benefits that are proportional to the earnings gained by individuals during their whole active life or the last part of it;
- the latter aims to achieve uniformity of individual returns by awarding pension benefits that depend on notional capital and life expectancy.

The two aims clash with one another. In fact the DC pursuit for uniformity of returns requires higher replacement rates for workers with lower wage growth. It also requires that the replacement rate increases with retirement age.

The DB scheme allows for some regressive redistribution while the ‘ortodox’ DC scheme described in sections 3.1 and 3.2 precludes any redistributive phenomena. One could think that this is not enough and that social solidarity suggests that flat careers should be advantaged.

As a matter of fact, DC schemes are compatible with redistribution mechanisms so as to temper their ‘individualistic’ nature. In fully transparent fashion, there could be provision for supplementing pensions that fall below some minimum and diminishing those above a given ceiling. Sustainability would be safeguarded by
properly coordinating these ceiling and floor levels so that the savings generated by the former would be enough to finance the additional spending for the latter.

One must still pose the question, however, of whether the pension system should be used for social solidarity, or whether it should be strictly an instrument for transferring income over time, leaving assistance to the needy to other welfare state institutions supported by general tax revenues.
Appendix

A.1. The DB scheme

A.1.1 The generational IRR

This section is devoted to prove a reformulation of Samuelson-Aaron theorem for an economy where heterogeneity of individual behaviour is allowed so that the wage bill growth rate can only be conceived as a generational IRR, which is not gained by individuals. The proof is given under the following assumptions:

- individuals live four years so that four generations overlap in each year;
- one can choose to work for two years, thus receiving an ‘ordinary’ pension for the other two, or to work just one year thus getting an ‘early retirement’ pension for three;
- there are two earnings levels denoted as ‘blue collar’ and ‘white collar’ wages;
- all workers are originally hired as blue-collar, so that the blue-collar wage is the entry wage;
- individual behaviour is stable for all cohorts, \( f_1 \) being the frequency with which workers choose early retirement, \( f_2 \) the frequency with which they work two years in blue-collar positions and \( f_3 \) the frequency with which they are promoted to white-collar status in the second year;
- both the blue-collar and the white-collar salaries grow exponentially at the rate \( \alpha > -1 \), which implies a constant percentage difference between the two, denoted as \( \beta \).
• the size of each cohort is $1 + \lambda$ times that of the preceding one, where $\lambda > -1$, so that employment grows at the same rate $\lambda$;

• finally, in accordance with formula (1), the first pension instalment is a share $n \cdot k$ of the last yearly wage ($n$ ranging from 1 to 2) and it grows at a constant indexation rate denoted as $\sigma$.

In order to prove that a balanced DB scheme yields a generational IRR equal to

\[ (10) \quad (1 + \alpha) \cdot (1 + \lambda) - 1, \]

observe that in any given year, hereafter referred to as ‘current year’, the wage bill, denoted as $W$, is the following sum of two components corresponding to job status (blue-collar and white-collar):

\[
W = \omega \cdot \left( N + f_2 \cdot \frac{N}{1 + \lambda} \right) + \omega \cdot (1 + \beta) \cdot \left( f_3 \cdot \frac{N}{1 + \lambda} \right),
\]

where $N$ is the size of the generation born in current year. Reordering, the wage bill becomes

\[ (11) \quad W = \frac{\omega \cdot N}{1 + \lambda} \left[ 1 + \lambda + f_2 + f_3 \cdot (1 + \beta) \right]. \]

Pension expenditure in the current year, hereinafter $P$, is the sum of the following 7 addends, distinguished by year of pension award, contribution history (early or ordinary retirement) and working status at retirement (blue- or white-collar worker):
Reordering, the pension expenditure becomes

\[
P = \frac{k \cdot w \cdot N}{(1 + \alpha) \cdot (1 + \lambda)} \left[ f_1 + \frac{2 \cdot f_2}{1 + \lambda} + \frac{2 \cdot f_3 \cdot (1 + \beta)}{(1 + \lambda)^2} + \frac{f_1 \cdot (1 + \sigma)}{(1 + \alpha) \cdot (1 + \lambda)} \right] + \frac{2 \cdot f_2 \cdot (1 + \sigma)}{(1 + \alpha) \cdot (1 + \lambda)^2} + \frac{2 \cdot f_3 \cdot (1 + \beta) \cdot (1 + \sigma)}{(1 + \alpha) \cdot (1 + \lambda)^2} + \frac{f_1 \cdot (1 + \sigma)^2}{(1 + \alpha)^2 \cdot (1 + \lambda)^2}.
\]

From (11) and (13) it follows that the equilibrium contribution rate is

\[
a = \frac{k \cdot \left\{ f_1 \cdot (1 + \Gamma + \Gamma^2) + \frac{2 \cdot [f_2 + f_3 \cdot (1 + \beta)]}{1 + \lambda} \right\} \cdot (1 + \Gamma)}{(1 + \alpha) \cdot \left[ 1 + \lambda + f_2 + f_3 \cdot (1 + \beta) \right]}
\]

where

\[
\Gamma = \frac{1 + \sigma}{(1 + \alpha) \cdot (1 + \lambda)}.
\]

The IRR, indicated as \( \pi \), to the generation born in the current year is the root of the following third-degree equation:

\[
A \cdot (1 + \pi)^3 + B \cdot (1 + \pi)^2 + C \cdot (1 + \pi) + D = 0
\]

where
Substituting (17) into (16), the latter takes the form

\[
a \cdot \left[ 1 + \left[ (1+\alpha) \cdot f_2 \right] \cdot \Pi^{-1} + \left[ (1+\alpha) \cdot (1+\beta) \cdot f_3 \right] \cdot \Pi^{-1} \right] = k \cdot \left[ f_1 \cdot \Pi^{-1} \cdot \left[ 1 + (1+\sigma) \cdot \Pi^{-1} + (1+\sigma)^2 \cdot \Pi^{-2} \right] + \left[ 2 \cdot (1+\alpha) \right] \cdot \Pi^{-2} \cdot \left[ f_2 + (1+\beta) \cdot f_1 + (1+\sigma) \cdot f_2 \cdot \Pi^{-1} + (1+\beta) \cdot (1+\sigma) \cdot f_3 \cdot \Pi^{-1} \right] \right],
\]

where

\[
\Pi = 1 + \pi.
\]

Substituting equation (14) into (18), one can finally check that the latter has the following positive solution

\[
\Pi = (1+\alpha) \cdot (1+\lambda).
\]

The existence of other non-negative solutions, implying meaningful (≥ −1) value for \(\pi\), is ruled out by scrutiny of the coefficients. In fact \(A\) is negative as the sum of individual contributions only, \(C\) and \(D\) are positive as the sum of pension benefits only, \(B\) is negative or positive depending on whether in the second year of the generation’s life contributions paid by those continuing to work are greater or less than pension benefits received by those already retired. In any case the vector of
coefficients (17) exhibits only one reversal of sign, so that Descartes’ rule guarantees that no other positive solutions can exist, while \( D > 0 \) excludes the null solution.\(^{31}\)

### A.1.2 Ranking individual IRRs

The IRR to any of the three types of behaviour allowed in the economy assumed in section A.1.1 is the meaningful root of an equation like (16) where, however, \( A, B, C, D \) represent individual rather than generational cash-flows. As in the case of (16), coefficients have only one reversal of sign, in that \( A \) is negative, \( C \) and \( D \) positive, and \( B \) positive or negative depending on whether early or ordinary retirement is elected. Hence one and only one meaningful solution (\( \pi \geq -1 \)) is guaranteed by Descartes’ rule and by \( D > 0 \) which rules out the solution \( \pi = -1 \).

As each behaviour produces its own coefficients, individual IRRs are generally unequal. The aim of this section is to rank such returns under the following reasonable assumptions:

\[
\begin{align*}
\alpha &> 0 \\
0 &< \sigma \leq \alpha \\
(21) \quad \beta &> 0 \\
a &> 0 \\
k &> 0
\end{align*}
\]

and, for the sake of generality, abandoning balance condition (14).

Each behaviour is associated with a quadruple of coefficients, as follows:

<table>
<thead>
<tr>
<th>Early retirement</th>
<th>Blue-collar career</th>
<th>White-collar career</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 = -a \cdot w )</td>
<td>( A_2 = -a \cdot w )</td>
<td>( A_3 = -a \cdot w )</td>
</tr>
<tr>
<td>( B_1 = k \cdot w )</td>
<td>( B_2 = -a \cdot w \cdot (1 + \alpha) )</td>
<td>( B_3 = -a \cdot w \cdot (1 + \alpha) \cdot (1 + \beta) )</td>
</tr>
<tr>
<td>( C_1 = k \cdot w \cdot (1 + \sigma) )</td>
<td>( C_2 = 2 \cdot k \cdot w \cdot (1 + \alpha) )</td>
<td>( C_3 = 2 \cdot k \cdot w \cdot (1 + \alpha) \cdot (1 + \beta) )</td>
</tr>
<tr>
<td>( D_1 = k \cdot w \cdot (1 + \sigma)^2 )</td>
<td>( D_2 = 2 \cdot k \cdot w \cdot (1 + \alpha) \cdot (1 + \sigma) )</td>
<td>( D_3 = 2 \cdot k \cdot w \cdot (1 + \alpha) \cdot (1 + \beta) \cdot (1 + \sigma) )</td>
</tr>
</tbody>
</table>

\(^{31}\) Actually, balance is not only a sufficient but also a necessary condition for the generational return to be the right hand side of (10). In fact equation (18) is linear in \( a \) so that at position (20) no
and therefore with one of the following polynomials in \( \Pi \):

\[
h_1(\Pi) = a \cdot \Pi^3 - k \cdot \Pi^2 - k \cdot (1 + \sigma) \cdot \Pi - k \cdot (1 + \sigma)^2
\]

\[
h_2(\Pi) = a \cdot \Pi^3 + a \cdot (1 + \alpha) \cdot \Pi^2 - 2 \cdot k \cdot (1 + \alpha) \cdot \Pi - 2 \cdot k \cdot (1 + \alpha) \cdot (1 + \sigma)
\]

\[
h_3(\Pi) = a \cdot \Pi^3 + a \cdot (1 + \alpha) \cdot (1 + \beta) \cdot \Pi^2 - 2 \cdot k \cdot (1 + \alpha) \cdot (1 + \beta) \cdot \Pi +
-2 \cdot k \cdot (1 + \alpha) \cdot (1 + \beta) \cdot (1 + \sigma).
\]

### A.1.2.1. \( \pi_2 \) versus \( \pi_3 \)

To rank the IRRs to blue-collar and white-collar careers, one can compare the positive zero of \( h_2(\Pi) \) with that of \( \frac{h_3(\Pi)}{1 + \beta} \) which has the same zeroes as \( h_3(\Pi) \). It is evident that

\[
h_2(0) = \frac{h_3(0)}{1 + \beta} = -2 \cdot k \cdot (1 + \alpha) \cdot (1 + \sigma) < 0
\]

and that

\[
h_2(\Pi) - \frac{h_3(\Pi)}{1 + \beta}(\Pi) = \left( a - \frac{a}{1 + \beta} \right) \cdot \Pi^3 > 0 \quad \forall \Pi > 0.
\]

Therefore the graphs of the two polynomials (in the meaningful interval) are as in figure 2 and the positive zero of \( h_2(\Pi) \), denoted as \( \Pi_2 \), is lower than that of \( \frac{h_3(\Pi)}{1 + \beta} \), denoted as \( \Pi_3 \). Hence \( \pi_3 > \pi_2 \).

\( \text{solutions (in } a \text{) other than (14) exist.} \)
A.1.2.2 $\pi_1$ versus $\pi_2$ and $\pi_3$

For $i$ which can take the values 2 and 3, to rank $\pi_1$ and $\pi_i$ one can compare the positive zero of $h_i(\Pi)$ with that of the polynomial

$$h_i(\Pi) = \frac{1+\sigma}{2(1+\alpha)B_i},$$

where

$$B_i = \begin{cases} 1 & \text{if } i = 2 \\ 1 + \beta & \text{if } i = 3. \end{cases}$$

It is evident that

$$h_1(0) = h_i(0) = \frac{1+\sigma}{2(1+\alpha)B_i} = -k \cdot (1+\sigma)^2$$

and that
\[ d_i(\Pi) = h_i(\Pi) \cdot \frac{(1+\sigma)}{2 \cdot (1+\alpha) \cdot B_i} - h_i(\Pi) = \]
\[ = \left[ \frac{a \cdot (1+\sigma)}{2 \cdot (1+\alpha) \cdot B_i} - a \right] \cdot \Pi^3 + \left[ \frac{a}{2} \cdot (1+\sigma) + k \right] \cdot \Pi^2. \]

Taking account of the ranges (21), the first coefficient of the polynomial \( d_i(\Pi) \) is negative and the second is positive. Therefore

\[ d_i(\Pi) > 0 \quad \text{if} \quad \Pi < \Pi_* \]
\[ d_i(\Pi) < 0 \quad \text{if} \quad \Pi > \Pi_* , \]

where

\[ \Pi_* = \frac{\left(1+\sigma + \frac{k}{2} \right)}{1 - \frac{1+\sigma}{2 \cdot (1+\alpha) \cdot B_i}} \]

is the only positive solution to the equation \( d_i(\Pi) = 0 \).

To compare \( \pi_i \) and \( \pi_j \) \((i = 2, 3)\) one must verify the sign of the polynomials \( h_i(\Pi) \) and (23) in correspondence with \( \Pi_* \). If the sign were positive, as shown in the left-hand panel of figure 3, the zero of \( h_i(\Pi) \) would be greater than that of \( h_i(\Pi) \) as a consequence of inequalities (25); otherwise, as in the right-hand panel, the opposite would hold.
Consider, therefore,

\[ h_i(\Pi_*) = a \cdot \left[ \Pi_* + B_i \cdot (1 + \alpha) \right] \cdot \Pi_*^2 - 2 \cdot k \cdot B_i \cdot (1 + \alpha) \cdot (\Pi_* + 1 + \sigma) \]

from which it follows that \( h_i(\Pi_*) > 0 \), and hence \( \Pi_1 > \Pi_i \), if

\[ \frac{\Pi_* + 1 + \alpha}{B_i} \cdot \frac{\Pi_*^2}{\Pi_* + 1 + \sigma} > \frac{2 \cdot k}{a} \cdot (1 + \alpha). \] (27)

Substituting (26) into (27) and setting \( \Phi = \frac{1 + \sigma}{1 + \alpha} \), finally, one gets

\[ \left[ \frac{\Phi}{2} + \frac{k}{a} \right]^2 \cdot \left( \frac{1 + \alpha}{\Phi a B_i} + \frac{k}{a B_i} \right) > \frac{2 \cdot k}{a} \cdot (1 + \alpha), \] (28)

which brings out several interesting aspects. If \( i = 2 \) (\( B_i = 1 \)), for \( \Phi = 1 \) inequality (28) reduces to the form

\[ \left( 1 + \alpha + \frac{2 \cdot k}{a} \right)^2 > \frac{2 \cdot k}{a} \cdot (1 + \alpha) \]
which is satisfied for every \( k/a \) and for every \( \alpha \). By continuity, (28) is therefore satisfied also for \( \Phi \) sufficiently near to 1, and hence for pension indexation sufficiently close to wage growth. Moreover, (28) holds also for any \( \alpha \) and \( \sigma \) if \( k/a \) is sufficiently large, i.e. if the DB scheme is sufficiently generous. This latter conclusion applies also to the case \( i = 3 \) (\( B_i = 1 + \beta \)).

**A.2. The defined-contribution scheme**

**A.2.1. The sustainable return**

This section is devoted to prove that setting the annual rate of return equal to the wage bill growth rate ensures sustainability (balance) of DC schemes provided that the employment growth rate be constant. The proof is given in the framework of the four-overlapping-generations economy already posited in section A.1.

Recalculating the addends of (12) on the basis of (3), pension expenditure becomes

\[
P = \frac{a \cdot w \cdot (1 + \pi) \cdot h(\delta, 3) \cdot f_1 \cdot N}{(1 + \alpha) \cdot (1 + \lambda)} + \left[ \frac{a \cdot w \cdot (1 + \pi) \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1)} + \frac{a \cdot w \cdot (1 + \pi)}{1 + \alpha} \right] \frac{h(\delta, 2) \cdot f_2 \cdot N}{(1 + \lambda)^2} + \\
\left[ \frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1) \cdot (1 + \lambda)^2} + \frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1) \cdot (1 + \alpha_2) \cdot (1 + \lambda)^2} \right] \frac{h(\delta, 2) \cdot (1 + \sigma) \cdot f_1 \cdot N}{(1 + \lambda)^3} + \\
\left[ \frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1) \cdot (1 + \alpha_2)} + \frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_2) \cdot (1 + \lambda)^2} \right] \frac{h(\delta, 2) \cdot (1 + \sigma) \cdot f_2 \cdot N}{(1 + \lambda)^3} + \\
\frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1) \cdot (1 + \alpha_2) \cdot (1 + \lambda)^3} \frac{h(\delta, 2) \cdot (1 + \sigma) \cdot f_1 \cdot N}{(1 + \lambda)^3} + \\
\frac{a \cdot w \cdot (1 + \pi \cdot (1 + \pi_1)}{(1 + \alpha) \cdot (1 + \alpha_1) \cdot (1 + \alpha_2) \cdot (1 + \lambda)^3} \frac{h(\delta, 2) \cdot (1 + \sigma) \cdot f_2 \cdot N}{(1 + \lambda)^3}
\]

where

- \( \pi, \alpha, \sigma \) denote, respectively, the values of the rate of return, wages growth, and indexation rate in the current year,
\( \pi, \alpha, \sigma \) \( (t = 1,2) \) denote the values of those same rates \( t \) years earlier,

\( h(\delta,3) \) and \( h(\delta,2) \) denote the conversion coefficients \( (6) \) for those retiring, respectively, in the second \( (m = 3) \) and the third \( (m = 2) \) year of life.

Taking account of \( (5) \), pension expenditure takes the form

\[
P = a \cdot w \cdot N \cdot \left\{ h(\delta,3) \cdot \theta \cdot f_1 + h(\delta,2) \cdot \left[ \theta \cdot \theta_1 + \frac{\theta}{1+\lambda} \right] \cdot f_2 + \left[ \theta \cdot \theta_1 + \frac{(1+\beta) \cdot \theta}{1+\lambda} \right] \cdot f_3 \right\} +
\]

\[
+ \frac{h(\delta,3) \cdot \theta \cdot \theta_1 \cdot \theta_2}{(1+\delta)^2} \cdot f_1,
\]

where the following positions hold:

\[
\theta = \frac{1+\pi}{(1+\alpha) \cdot (1+\lambda)}
\]

\[
\theta_1 = \frac{1+\pi_1}{(1+\alpha_1) \cdot (1+\lambda)}
\]

\[
\theta_2 = \frac{1+\pi_2}{(1+\alpha_2) \cdot (1+\lambda)}
\]

Grouping by common factor, from \( (29) \) it follows

\[
P = a \cdot w \cdot N \cdot h(\delta,3) \cdot \left[ \theta + \frac{\theta \cdot \theta_1}{1+\delta} + \frac{\theta \cdot \theta_1 \cdot \theta_2}{(1+\delta)^2} \right] \cdot f_1 +
\]

\[
+ h(\delta,2) \cdot \left[ \theta \cdot \theta_1 + \frac{\theta}{1+\lambda} \right] \cdot f_2 + \left[ \theta \cdot \theta_1 + \frac{(1+\beta) \cdot \theta}{1+\lambda} \right] \cdot f_3 \right\} +
\]

\[
+ h(\delta,2) \cdot \left[ \theta \cdot \theta_1 + \frac{(1+\beta) \cdot \theta}{1+\lambda} \right] \cdot f_2 + \left[ \theta \cdot \theta_1 + \frac{(1+\beta) \cdot \theta}{1+\lambda} \right] \cdot f_3 \right\}.
\]

If the rate of return is equal to the growth of the wage bill, parameters \( (30) \) reduce to 1, so that \( (31) \) takes the further form
\[ P = a \cdot w \cdot N \cdot \left\{ h(\delta, 3) \cdot \left[ 1 + (1 + \delta)^{-1} + (1 + \delta)^{-2} \right] \cdot f_1 + \\
+ h(\delta, 2) \cdot \left[ 1 + (1 + \delta)^{-1} \right] \cdot \left[ \left( 1 + \frac{1}{1 + \lambda} \right) \cdot f_2 + \left( 1 + \frac{1 + \beta}{1 + \lambda} \right) \cdot f_3 \right] \right\}. \]

(32)

In conformity with (6), the conversion coefficients are

\[ h(\delta, 3) = \left[ 1 + (1 + \delta)^{-1} + (1 + \delta)^{-2} \right]^{-1} \]

\[ h(\delta, 2) = \left[ 1 + (1 + \delta)^{-1} \right]^{-1}, \]

so that (32) becomes

\[ P = a \cdot w \cdot N \cdot \frac{1 + \lambda + f_2 + f_3 \cdot (1 + \beta)}{1 + \lambda}. \]

Taking account of (11), it finally follows that \( P = a \cdot W. \)

The theorem has an interesting corollary: if the return awarded by a DC scheme equals the wage bill growth rate, then pension expenditure is independent of \( \delta \); but if the return exceeds the wage bill growth rate, from (30) and (31) it follows that \( \frac{\partial P}{\partial \delta} < 0 \) so that annual deficits could at least be limited by choosing high values of \( \delta \).

**A.2.2. The effects of exogenous shocks on \( \lambda \)**

The table attached shows a dynamic simulation of the effects produced on DC schemes by a one-time increment in the parameter \( \lambda \) (the rate of growth of new workers). The simulation is run in the usual framework of four-overlapping-generations. The shock in \( \lambda \) is administered in year 1. The table covers the years from \(-5\) to \(+6\).
The simulation shows the temporary formation (years 2 and 3) of surpluses after which balance is automatically restored (year 4). From there on (starting in year 5) expenditure and revenue both rise at the new rate of increase of the wage bill.

Symmetrically, a reduction in $\lambda$ would generate temporary deficits.

---

32 Actually, that the rate of return be equal to the wage bill growth is not only a sufficient but also a necessary condition for the NDC scheme to be sustainable. In fact, from (30) and (31) it follows that $\partial P / \partial \pi > 0$ so that the sustainability condition $P = a \cdot W$ will at most admit one solution in $\pi$. 
The effects of a rise in the rate of increase in number of new workers

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<tr>
<td>1.1. $\alpha$ from year 1</td>
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<td>1.2. $\chi$ up to year 0</td>
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<tr>
<td>1.3. $\chi$ from year 1</td>
</tr>
</tbody>
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<table>
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<th>2. RESULTS</th>
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<td>2.1. Time</td>
</tr>
<tr>
<td>-5</td>
</tr>
<tr>
<td>669</td>
</tr>
<tr>
<td>43,0%</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>2.2. Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>496</td>
</tr>
<tr>
<td>43,0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3. Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.369</td>
</tr>
<tr>
<td>+43,0%</td>
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</tbody>
</table>

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<thead>
<tr>
<th>2.4. Revenue/expenditure</th>
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<td>1</td>
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<thead>
<tr>
<th>2.5. Expenditure for new instalsments</th>
</tr>
</thead>
<tbody>
<tr>
<td>660</td>
</tr>
<tr>
<td>43,0%</td>
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<table>
<thead>
<tr>
<th>2.6. Expenditure for continuing pensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>496</td>
</tr>
<tr>
<td>43,0%</td>
</tr>
</tbody>
</table>

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<tr>
<th>3. DETAILS</th>
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<td>3.1. Calculation of the Wage bill</td>
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<tr>
<td>3.1.1. Workers in 1st year</td>
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<td>3.2. Return</td>
</tr>
<tr>
<td>43,0%</td>
</tr>
<tr>
<td>3.3. Indexation</td>
</tr>
<tr>
<td>19.2%</td>
</tr>
</tbody>
</table>

| 3.4. Calculation of Expenditure |
| 3.4.1. Early retirement pensions |
| 3.4.1.1. awarded in year t | 49 | 69 | 99 | 142 | 214 | 345 | 539 | 841 | 1.311 | 2.046 |
| 3.4.1.2. awarded in year t-1 | 58 | 83 | 118 | 179 | 279 | 449 | 701 | 1.093 | 1.705 |
| 3.4.1.3. awarded in year t-2 | 69 | 99 | 149 | 232 | 362 | 584 | 911 | 1.421 |
| 3.4.2. Ordinary pensions |
| 3.4.2.1. awarded in year t | 426 | 610 | 872 | 1.316 | 2.001 | 3.178 | 4.958 | 7.734 | 12.066 |
| 3.4.2.1. awarded in year t-1 | 508 | 726 | 1.097 | 1.711 | 2.602 | 4.132 | 6.445 | 10.055 |

(*) The value of this parameter derives from the value chosen for $\delta$
References


Bröms J. (1990), *Ur askan av ATP*, SACO, Stockholm.


